W1L7 - SEPARABLE DIFFERENTIAL EQUATIONS

Recall that: $\int$ can solve some diff eq.

Idea: group "y" + dy on one side and "x" + dx on the other and integrate both sides

Works for: $\frac{dy}{dx} = f(x,y)$ when $f(x,y)$ can be written as a product

Notes:
1. keep constants on the "x"-side
2. solve for "y" explicitly in solution if possible

EX

$$\frac{dy}{dx} + 2xy = 0$$

$$-2xy \quad -2xy$$

$$\frac{dy}{dx} = -2xy$$

$$\frac{1}{y}\frac{dy}{dx} = -2x$$

$$\int \frac{1}{y}\,dy = \int -2x\,dx$$

we can show both C's (from each side of the equ.) as one C

$$\ln|y| = -x^2 + C_1$$

$$e^{\ln|y|} = e^{-x^2+C_1}$$

$$|y| = e^{-x^2+C_1}$$

$$|y| = e^{-x^2}\cdot e^{C_1}$$

$$|y| = e^{C_1}\cdot e^{-x^2}$$

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

$$y = \pm e^{C_1}\cdot e^{-x^2}$$

Let $C = \pm e^{C_1}$

$$y = Ce^{-x^2}$$ ← General Solution

EX

$$\frac{dy}{dx} + 2xy^2 = 0$$

$$\frac{dy}{dx} = -2x \cdot y^2$$

$$\int \frac{1}{y^2}\,dy = \int -2x\,dx$$

$$\int y^{-2}\,dy = \int -2x\,dx$$

$$-y^{-1} = -x^2 + C_1$$

$$-\frac{1}{y} = -x^2 + C_1$$

$$\frac{1}{y} = x^2 - C_1$$

$$y = \frac{1}{x^2 - C} \quad \leftarrow \text{General Solution}$$

EX

$$\frac{dy}{dx} = y\sin(x)$$

$$\int \frac{1}{y}\,dy = \int \sin(x)\,dx$$

$$\ln|y| = -\cos(x) + C_1$$

$$|y| = e^{-\cos(x) + C_1}$$

$$|y| = e^{C_1} \cdot e^{-\cos(x)}$$

LET $C = \pm e^{C_1}$

$$y = \pm e^{C_1} \cdot e^{-\cos(x)}$$

$$y = Ce^{-\cos(x)} \quad \leftarrow \text{General Solution}$$

EX

$$(1+x)\frac{dy}{dx} = 4y$$

$$\int \frac{1}{y}\,dy = \int \frac{4}{1+x}\,dx$$

$$\int \frac{1}{1+x} \Rightarrow \int \frac{1}{u}\,du \Rightarrow \ln|1+x|$$

$u = 1+x$

$du = dx$

$$\ln|y| = 4\ln|1+x| + C_1$$

$$\ln|y| = \ln(1+x)^4 + C_1$$

$$|y| = e^{\ln(1+x)^4 + C_1}$$

$$|y| = e^{C_1} \cdot e^{\ln(1+x)^4} \Rightarrow |y| = e^{C_1}(1+x)^4 \Rightarrow y = \pm e^{C_1}(1+x)^4$$

$$y = C(1+x)^4 \quad \leftarrow \text{General Solution}$$

EX

$$2\sqrt{x}\,\frac{dy}{dx} = \sqrt{1-y^2}$$

$$2\sqrt{x}\,\frac{1}{dx} = \sqrt{1-y^2}\,\frac{1}{dy}$$

$$\frac{1}{2\sqrt{x}}\,dx = \frac{1}{\sqrt{1-y^2}}\,dy$$

$$\int \frac{1}{\sqrt{1-y^2}}\,dy = \int \frac{1}{2\sqrt{x}}\,dx$$

$$\sin^{-1}(y) = \int \frac{1}{2} x^{-1/2}\,dx$$

$$\vdots \qquad = \frac{1}{2}\,\frac{x^{1/2}}{1/2} + C_1$$

$$\sin^{-1}(y) = \sqrt{x} + C_1$$

$$y = \sin(\sqrt{x} + C)$$